

Five short lemmas in Banach spaces

QINGPING ZENG

ABSTRACT.

Consider a commutative diagram of bounded linear operators between Banach spaces

$$\begin{array}{ccccccccc} 0 & \longrightarrow & X & \xrightarrow{J} & Y & \xrightarrow{Q} & Z & \longrightarrow & 0 \\ & & A \downarrow & & B \downarrow & & C \downarrow & & \\ 0 & \longrightarrow & X & \xrightarrow{J} & Y & \xrightarrow{Q} & Z & \longrightarrow & 0 \end{array}$$

with exact rows. In what ways are the spectral and local spectral properties of B related to those of the pairs of operators A and C ? In this paper, we give our answers to this general question using tools from local spectral theory.

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FUJIAN AGRICULTURE AND FORESTRY UNIVERSITY
COLLEGE OF COMPUTER AND INFORMATION SCIENCES
350002 FUZHOU, P. R. CHINA
E-mail address: zqpping2003@163.com

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