Five short lemmas in Banach spaces

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ABSTRACT.

Consider a commutative diagram of bounded linear operators between Banach spaces

$$0 \longrightarrow X \stackrel{J}{\longrightarrow} Y \stackrel{Q}{\longrightarrow} Z \longrightarrow 0$$

$$A\downarrow \qquad B\downarrow \qquad C\downarrow$$

$$0 \longrightarrow X \stackrel{J}{\longrightarrow} Y \stackrel{Q}{\longrightarrow} Z \longrightarrow 0$$

with exact rows. In what ways are the spectral and local spectral properties of B related to those of the pairs of operators A and C? In this paper, we give our answers to this general question using tools from local spectral theory.

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