

## Fixed points of mappings defined on spaces with distance

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### ABSTRACT.

In the present article we study distinct metrical structures guaranteeing the existence of fixed points for a given mapping (Propositions 3.4 and 5.9, Theorems 4.1 and 7.3, Corollaries 2.1, 3.3, 4.4, 4.7, 5.10, 6.12, 6.13). Some examples are proposed (Examples 1.4, 4.9, 6.12).

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