

## A generalized Birkhoff–Young quadrature formula

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### ABSTRACT.

A generalized  $(4n + 1)$ -point Birkhoff–Young quadrature of interpolatory type with the maximal degree of precision for numerical integration of analytic functions is derived. An explicit form of the node polynomial of such kind of quadratures is obtained. Special cases and an example are presented.

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