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About division quaternion algebras and division symbol algebras

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ABSTRACT.

In this paper, we find a class of division quaternion algebras over the field $\mathbb{Q}(i)$ and a class of division symbol algebras over a cyclotomic field.

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REFERENCES

[1] Alexandru, V. and Gosoniu, N. M., *Elements of Number Theory* (in Romanian), Ed. Bucharest University, 1999
[2] Alsina, M. and Bayer, P., *Quaternion Orders, Quadratic Forms and Shimura Curves*, CRM Monograph Series, **22**, American Mathematical Society, 2004
[3] Borevich, Z. I. and Shafarevich, I. R., *Number Theory*, Academic Press Inc, New York, 1966
[4] Brown, E. and Parry, C. J., *The imaginary bicyclic biquadratic fields with class - number* 1, J. Reine Angew Math., **226** (1974), 118–120
[5] Elman, R. and Lam, T. Y., *Classification theorems for quadratic forms over fields*, Comment. Math. Helv., **49** (1974), 373–381
[6] Flaut, C., *Division algebras with dimension* $2^t, t \in \mathbb{N}$, An. Sti. U. Ovid. Co-Mat., **13** (2005), No. 2, 31–38
[7] Flaut, C. and Savin, D.,*Some properties of symbol algebras of degree three*, Math. Rep. (Bucur.), **16(66)** (2014), No. 3, 443–463
[8] Flaut, C., Savin, D., *Some examples of division symbol algebras of degree* 3 *and* 5, Carpathian J. Math., **31** (2015), No. 2, 197–204
[9] Gille, P. and Szamuely, T., *Central Simple Algebras and Galois Cohomology*, Cambridge University Press, 2006
[10] Kyrchei, I., *The column and row immanants over a split quaternion algebra*, Adv. Appl. Clifford Algebr, **25** (2015), No. 3, 611–619
[11] Kohel, D., *Quaternion algebras*, echidna.maths.usyd.edu.au/kohel/alg/doc/ AlgQuat.pdf
[12] Kohel, D., *Hecke module structure of quaternions*, Proceedings of Class Field Theory - Centenary and Prospect (Tokyo, 1998), K. Miyake, ed., Advanced Studies in Pure Mathematics, **30**, 177–196, 2000
[13] Kohel, D., Lauter, K., Petit, C. and Tignol, J.-P., *On the quaternion ℓ-isogeny path problem*, LMS J. Comput. Math., **17** (2014), 418–432

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[14] Lam, T. Y., *Introduction to Quadratic Forms over Fields*, AMS, 2004
[15] Lang, S., *Algebra*, Springer-Verlag, 2002
[16] Ledet, A., *Brauer Type Embedding Problems*, American Mathematical Society, 2005
[17] Lemmermeyer, F., *Reciprocity laws, from Euler to Eisenstein*, Springer, 2000
[18] Marcus, D., *Number fields*, Universitext, 1995
[19] Milne, J. S., *Class Field Theory*, http://www.math.lsa.umich.edu/ jmilne
[20] Milnor, J., *Introduction to Algebraic K-Theory*, , Annals of Mathematics Studies, Princeton Univ. Press, 1971
[21] Olteanu, G., *Baer-Galois connections and applications*, Carpathian J. Math., **30** (2014), No. 2, 225–229
[22] Pierce, R. S., *Associative Algebras*, Springer Verlag, 1982
[23] Ramirez, J. L., *Some combinatorial properties of the k-Fibonacci and the k-Lucas quaternions*, An. tiin. Univ. "Ovidius" Constana Ser. Mat., **23** (2015), No. 2, 201–212
[24] Savin, D., Flaut, C. and Ciobanu, C., *Some properties of the symbol algebras*, Carpathian J. Math. , **25** (2009), No. 2, 239–245 (arXiv:0906.2715)
[25] Savin, D., *About some split central simple algebras*, An. Stiin. Univ. "'Ovidius" Constanta, Ser. Mat, **22** (2014), No. 1, 263–272
[26] Vigneras, M. F., *Arithmetique des alg'ebres de quaternions*, Lecture Notes in Math., No. 800, Springer, 1980
[27] Voight, J., *The Arithmetic of Quaternion Algebras*, available on the website: http://www.math. dartmouth.edu/ jvoight/crmquat/book/quat-modforms-041310.pdf, 2010

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