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A path convergence theorem and construction of fixed points for nonexpansive mappings in certain Banach spaces

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ABSTRACT.

In this paper, we introduce a new iterative process to approximate fixed points of nonexpansive maps in real Banach spaces having weakly continuous duality map and establish strong convergence theorems for the proposed iterative process. There is no compactness assumption on K or on T. Our results improve important recent results.

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