

## Rings in which nilpotents form a subring

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### ABSTRACT.

Let  $R$  be a ring with the set of nilpotents  $\text{Nil}(R)$ . We prove that the following are equivalent: (i)  $\text{Nil}(R)$  is additively closed, (ii)  $\text{Nil}(R)$  is multiplicatively closed and  $R$  satisfies Köthe's conjecture, (iii)  $\text{Nil}(R)$  is closed under the operation  $x \circ y = x + y - xy$ , (iv)  $\text{Nil}(R)$  is a subring of  $R$ . Some applications and examples of rings with this property are given, with an emphasis on certain classes of exchange and clean rings.

**Acknowledgements.** The author would like to thank Nik Stopar for helpful discussions and Pace P. Nielsen for useful comments on the previous version of this paper.

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Received: 05.10.2015; In revised form: 29.12.2015; Accepted: 05.01.2016  
2010 *Mathematics Subject Classification.* 16N40, 16U99.

Key words and phrases. *Nilpotent, NR ring, Armendariz ring, exchange ring, strongly nil clean ring.*