

Dedicated to Professor Emeritus Ioan A. Rus on the occasion of his 80<sup>th</sup> anniversary

## Retraction method in fixed point theory for monotone nonexpansive mappings

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### ABSTRACT.

In this paper we study the properties of the common fixed points set of a commuting family of monotone nonexpansive mappings in Banach spaces endowed with a graph. In particular, we prove that under certain conditions, this set is a monotone nonexpansive retract.

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