

Dedicated to Professor Emeritus Ioan A. Rus on the occasion of his 80<sup>th</sup> anniversary

## Fixed point theorems for nonself Kannan type contractions in Banach spaces endowed with a graph

LASZLO BALOG<sup>1</sup> and VASILE BERINDE<sup>1,2</sup>

### ABSTRACT.

Let  $K$  be a non-empty closed subset of a Banach space  $X$  endowed with a graph  $G$ . The main result of this paper is a fixed point theorem for nonself Kannan  $G$ -contractions  $T : K \rightarrow X$  that satisfy Rothe's boundary condition, i.e.,  $T$  maps  $\partial K$  (the boundary of  $K$ ) into  $K$ . Our new results are extensions of recent fixed point theorems for self mappings on metric spaces endowed with a partial order and also of various fixed point theorems for self and nonself mappings on Banach spaces or convex metric spaces.

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### REFERENCES

- [1] Abbas, M., Ali, B. and Petruşel, G., *Fixed points of set-valued contractions in partial metric spaces endowed with a graph*, Carpathian J. Math., **30** (2014), No. 2, 129–137
- [2] Agarwal, R. P., El-Gebeily, M. A. and O'Regan, D., *Generalized contractions in partially ordered metric spaces*, Appl. Anal., **87** (2008), 1–8
- [3] Alghamdi, Maryam A., Berinde, V. and Shahzad, N., *Fixed points of multi-valued non-self almost contractions*, J. Appl. Math., Volume **2013**, Article ID 621614, 6 pp.
- [4] Alghamdi, Maryam A., Berinde, V. and Shahzad, N., *Fixed points of non-self almost contractions*, Carpathian J. Math., **33** (2014), No. 1, 1–8
- [5] Ariza-Ruiz, D. and Jiménez-Melado, A., *A continuation method for weakly Kannan maps*, Fixed Point Theory Appl., **2010**, Art. ID 321594, 12 pp.
- [6] Assad, N. A. *On a fixed point theorem of Iséki*, Tamkang J. Math., **7** (1976), No. 1, 19–22
- [7] Assad, N. A. *On a fixed point theorem of Kannan in Banach spaces*, Tamkang J. Math., **7** (1976), No. 1, 91–94
- [8] Assad, N. A., *On some nonself nonlinear contractions*, Math. Japon., **33** (1988), No. 1, 17–26
- [9] Assad, N. A., *On some nonself mappings in Banach spaces*, Math. Japon., **33** (1988), No. 4, 501–515
- [10] Assad, N. A., *Approximation for fixed points of multivalued contractive mappings*, Math. Nachr., **139** (1988), 207–213

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Corresponding author: Vasile Berinde; vasile.berinde@gmail.com

- [11] Assad, N. A., *A fixed point theorem in Banach space*, Publ. Inst. Math. (Beograd) (N.S.), **47** (1990), No. 61, 137–140
- [12] Assad, N. A., *A fixed point theorem for some non-self-mappings*, Tamkang J. Math., **21** (1990), No. 4, 387–393
- [13] Assad, N. A. and Kirk, W. A., *Fixed point theorems for set-valued mappings of contractive type*, Pacific J. Math., **43** (1972), 553–562
- [14] Assad, N. A. and Sessa, S., *Common fixed points for nonself compatible maps on compacta*, Southeast Asian Bull. Math., **16** (1992), No. 2, 91–95
- [15] Berinde, V., *A common fixed point theorem for nonself mappings*, Miskolc Math. Notes, **5** (2004), No. 2, 137–144
- [16] Berinde, V., *Approximation of fixed points of some nonself generalized  $\phi$ -contractions*, Math. Balkanica (N.S.), **18** (2004), No. 1-2, 85–93
- [17] Berinde, V., *Iterative Approximation of Fixed Points*, 2nd Ed., Springer Verlag, Berlin Heidelberg New York, 2007
- [18] Berinde, V. and Păcurar, M., *Fixed point theorems for nonself single-valued almost contractions*, Fixed Point Theory, **14** (2013), No. 2, 301–312
- [19] Berinde, V. and Păcurar, M., *The contraction principle for nonself mappings on Banach spaces endowed with a graph*, J. Nonlinear Convex Anal., **16** (2015), No. 9, 1925–1936
- [20] Berinde, V. and Păcurar, M., *A constructive approach to coupled fixed point theorems in metric spaces*, Carpathian J. Math., **31** (2015), No. 3, 277–287
- [21] Berinde, V. and Petric, M. A., *Fixed point theorems for cyclic non-self single-valued almost contractions*, Carpathian J. Math., **31** (2015), No. 3, 289–296
- [22] Bojor F., *Fixed point of  $\varphi$ -contraction in metric spaces endowed with a graph*, Ann. Univ. Craiova, Math. Comput., Sci. Ser., **37** (2010), No. 4, 85–92
- [23] Bojor, F., *Fixed points of Bianchini mappings in metric spaces endowed with a graph*, Carpathian J. Math., **28** (2012), No. 2, 207–214
- [24] Bojor, F., *Fixed points of Kannan mappings in metric spaces endowed with a graph*, An. Stiint. Univ. "Ovidius" Constanta Ser. Mat., **20** (2012), No. 1, 31–40
- [25] Bojor, F., *Fixed point theorems for Reich type contractions on metric spaces with a graph*, Nonlinear Anal., **75** (2012), No. 9, 3895–3901
- [26] Bojor, F., *Fixed point theorems in in metric spaces endowed with a graph* (in Romanian), PhD Thesis, North University of Baia Mare, 2012
- [27] Caristi, J., *Fixed point theorems for mappings satisfying inwardness conditions*, Trans. Amer. Math. Soc., **215** (1976), 241–251
- [28] Caristi, J., *Fixed point theory and inwardness conditions*, Applied nonlinear analysis (Proc. Third Internat. Conf., Univ. Texas, Arlington, Tex., 1978), pp. 479–483, Academic Press, New York-London, 1979
- [29] Chatterjea, S. K., *Fixed-point theorems*, C. R. Acad. Bulgare Sci., **25** (1972) 727–730
- [30] Chifu, C. and Petrușel, G., *Generalized contractions in metric spaces endowed with a graph*, Fixed Point Theory Appl., 2012, 2012:161, 9 pp.
- [31] Cho, S.-H., *A fixed point theorem for a Ćirić-Berinde type mapping in orbitally complete metric spaces*, Carpathian J. Math., **30** (2014), No. 1, 63–70
- [32] Choudhury, B. S., Das, K. and Bhandari, S. K., *Cyclic contraction of Kannan type mappings in generalized Menger space using a control function*, Azerb. J. Math., **2** (2012), No. 2, 43–55
- [33] Ćirić, Lj. B., *A remark on Rhoades' fixed point theorem for non-self mappings*, Internat. J. Math. Math. Sci., **16** (1993), No. 2, 397–400
- [34] Ćirić, Lj. B., *Quasi contraction non-self mappings on Banach spaces*, Bull. Cl. Sci. Math. Nat. Sci. Math., (1998), No. 23, 25–31
- [35] Ćirić, Lj. B., Ume, J. S., Khan, M. S. and Pathak, H. K., *On some nonself mappings*, Math. Nachr., **251** (2003), 28–33
- [36] Eisenfeld, J. and Lakshmikantham, V., *Fixed point theorems on closed sets through abstract cones*, Appl. Math. Comput., **3** (1977), No. 2, 155–167
- [37] Filip, A.-D., *Fixed point theorems for multivalued contractions in Kasahara spaces*, Carpathian J. Math., **31** (2015), No. 2, 189–196
- [38] Gabeleh, M., *Existence and uniqueness results for best proximity points*, Miskolc Math. Notes, **16** (2015), No. 1, 123–131
- [39] Hussain, N., Salimi, P. and Vetro, P., *Fixed points for  $\alpha$ - $\psi$ -Suzuki contractions with applications to integral equations*, Carpathian J. Math., **30** (2014), No. 2, 197–207
- [40] Jachymski, J., *The contraction principle for mappings on a metric space with a graph*, Proc. Amer. Math. Soc., **136** (2008), No. 4, 1359–1373

- [41] Kannan, R., *Some results on fixed points*, Bull. Calcutta Math. Soc., **10** (1968), 71–76
- [42] Kikkawa, M. and Suzuki, T., *Some similarity between contractions and Kannan mappings. II*, Bull. Kyushu Inst. Technol. Pure Appl. Math., (2008), No. 55, 1–13
- [43] Kikkawa, M. and Suzuki, T., *Some similarity between contractions and Kannan mappings*, Fixed Point Theory Appl., 2008, Art. ID 649749, 8 pp.
- [44] Kirk, W. A., Srinivasan, P. S. and Veeramani P., *Fixed points for mappings satisfying cyclical contractive conditions*, Fixed Point Theory, **4** (2003), No. 1, 79–89
- [45] Meszaros, J., *A comparison of various definitions of contractive type mappings*, Bull. Calcutta Math. Soc., **84** (1992), No. 2, 167–194
- [46] Nicolae, A., O'Regan, D. and Petruşel, A., *Fixed point theorems for singlevalued and multivalued generalized contractions in metric spaces endowed with a graph*, Georgian Math. J., **18** (2011), No. 2, 307–327
- [47] Nieto, J. J. and Rodriguez-Lopez, R., *Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations*, Order **22** (2005), No. 3, 223–239 (2006)
- [48] Nieto, J. J. and Rodriguez-Lopez, R., *Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations*, Acta. Math. Sin., (Engl. Ser.) **23** (2007), No. 12, 2205–2212
- [49] Nieto, Juan J., Pouso, Rodrigo L. and Rodriguez-Lopez, R., *Fixed point theorems in ordered abstract spaces*, Proc. Amer. Math. Soc., **135** (2007), No. 8, 2505–2517
- [50] Panja, C. and Samanta, S. K., *On determination of a common fixed point*, Indian J. Pure Appl. Math., **11** (1980), No. 1, 120–127
- [51] Păcurar, M., *Approximating common fixed points of Prešić-Kannan type operators by a multi-step iterative method*, An. Ştiinţ. Univ. "Ovidius" Constanţa Ser. Mat., **17** (2009), No. 1, 153–168
- [52] Păcurar, M., *Iterative Methods for Fixed Point Approximation*, Risoprint, Cluj-Napoca, 2010
- [53] Păcurar, M., *A multi-step iterative method for approximating fixed points of Prešić-Kannan operators*, Acta Math. Univ. Comen. New Ser., **79** (2010), No. 1, 77–88
- [54] Păcurar, M., *A multi-step iterative method for approximating common fixed points of Prešić-Rus type operators on metric spaces*, Stud. Univ. Babeş-Bolyai Math., **55** (2010), No. 1, 149–162
- [55] Păcurar, M., *Fixed points of almost Prešić operators by a k-step iterative method*, An. Ştiinţ. Univ. Al. I. Cuza Iaşi, Ser. Noua, Mat., **57** (2011), Supliment 199–210
- [56] Petric, M., *Some results concerning cyclical contractive mappings*, Gen. Math., **18** (2010), No. 4, 213–226
- [57] Petric, M., *Best proximity point theorems for weak cyclic Kannan contractions*, Filomat, **25** (2011), No. 1, 145–154
- [58] Petruşel, A. and Rus, I. A., *Fixed point theorems in ordered L-spaces*, Proc. Amer. Math. Soc., **134** (2006), No. 2, 411–418
- [59] Ran, A. C. M. and Reurings, M. C. B., *A fixed point theorem in partially ordered sets and some applications to matrix equations*, Proc. Amer. Math. Soc., **132** (2004), No. 5, 1435–1443
- [60] Rhoades, B. E., *A comparison of various definitions of contractive mappings*, Trans. Amer. Math. Soc., **226** (1977) 257–290
- [61] Rhoades, B. E., *A fixed point theorem for some non-self-mappings*, Math. Japon., **23** (1978/79), No. 4, 457–459
- [62] Rhoades, B. E., *Contractive definitions revisited*, Contemporary Mathematics, **21** (1983), 189–205
- [63] Rhoades, B. E., *Contractive definitions and continuity*, Contemporary Mathematics, **72** (1988), 233–245
- [64] Rus, I. A., *Principles and Applications of the Fixed Point Theory* (in Romanian), Editura Dacia, Cluj-Napoca, 1979
- [65] Rus, I. A., *Generalized contractions*, Seminar on Fixed Point Theory, **3** (1983), 1–130
- [66] Rus, I. A., *Generalized Contractions and Applications*, Cluj University Press, Cluj-Napoca, 2001
- [67] Rus, I. A., *Picard operators and applications*, Sci. Math. Jpn., **58** (2003), No. 1, 191–219
- [68] Rus, I. A., *Private communication* (2015)
- [69] Rus, I. A., Petruşel, A. and Petruşel, G., *Fixed Point Theory*, Cluj University Press, Cluj-Napoca, 2008
- [70] Samanta, S. K., *Fixed point theorems for non-self-mappings*, Indian J. Pure Appl. Math., **15** (1984), No. 3, 221–232
- [71] Samanta, S. K., *Fixed point theorems for Kannan maps in a metric space with some convexity structure*, Bull. Calcutta Math. Soc., **80** (1988), No. 1, 58–64
- [72] Samanta, C. and Samanta, S. K., *Fixed point theorems for multivalued non-self mappings*, Zb. Rad. Prirod.-Mat. Fak. Ser. Mat., **22** (1992), No. 1, 11–22
- [73] Shukla, S. and Abbas, M., *Fixed point results of cyclic contractions in product spaces*, Carpathian J. Math., **31** (2015), No. 1, 119–126
- [74] Sun, Y. I., Su, Y. F. and Zhang, J. L., *A new method for the research of best proximity point theorems of nonlinear mappings*, Fixed Point Theory Appl., **2014**, 2014:116, 18 pp.
- [75] Ume, J. S., *Fixed point theorems for Kannan-type maps*, Fixed Point Theory Appl., **2015**, 2015:38, 13 pp.

- [76] Zhang, J. L. and Su, Y. F., *Best proximity point theorems for weakly contractive mapping and weakly Kannan mapping in partial metric spaces*, *Fixed Point Theory Appl.*, **2014**, 2014:50, 8 pp.

<sup>1</sup>DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE  
NORTH UNIVERSITY CENTER AT BAIA MARE  
TECHNICAL UNIVERSITY OF CLUJ-NAPOCA  
VICTORIEI 76, 430122 BAIA MARE , ROMANIA  
*E-mail address*: vberinde@ubm.ro  
*E-mail address*: balog\_58@yahoo.com

<sup>2</sup>DEPARTMENT OF MATHEMATICS AND STATISTICS  
KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS DHAHRAN, SAUDI ARABIA  
*E-mail address*: vasile.berinde@gmail.com