

Dedicated to Professor Emeritus Ioan A. Rus on the occasion of his 80<sup>th</sup> anniversary

## Fixed point approximation of Prešić nonexpansive mappings in product of $CAT(0)$ spaces

HAFIZ FUKHAR-UD-DIN<sup>1,2</sup>, VASILE BERINDE<sup>3,1</sup> and ABDUL RAHIM KHAN<sup>1</sup>

### ABSTRACT.

We obtain a fixed point theorem for Prešić nonexpansive mappings on the product of  $CAT(0)$  spaces and approximate this fixed points through Ishikawa type iterative algorithms under relaxed conditions on the control parameters. Our results are new in the literature and are valid in uniformly convex Banach spaces.

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Corresponding author: Vasile Berinde; [vberinde@ubm.ro](mailto:vberinde@ubm.ro)

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<sup>1</sup>DEPARTMENT OF MATHEMATICS AND STATISTICS  
 KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS  
 DHAHRAM, KINGDOM OF SAUDI ARABIA  
 E-mail address: hfdin@kfupm.edu.sa; arahim@kfupm.edu.sa

<sup>2</sup>DEPARTMENT OF MATHEMATICS  
 THE ISLAMIA UNIVERSITY OF BAHAWALPUR  
 BAHAWALPUR 63100, PAKISTAN

<sup>3</sup>DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE  
 NORTH UNIVERSITY CENTER AT BAIA MARE  
 TECHNICAL UNIVERSITY OF CLUJ-NAPOCA  
 VICTORIEI 76, 430122 BAIA MARE, ROMANIA  
 E-mail address: vberinde@ubm.ro