

Dedicated to Professor Emeritus Ioan A. Rus on the occasion of his 80th anniversary

Coupled fixed points and α -dense curves

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ABSTRACT.

We present a new iterative method, based on the so called α -dense curves, to approximate coupled fixed points of nonexpansive mappings. Compactness condition on the mapping or its domain of definition is necessary. As application, we construct a sequence which converges to a solution of certain system of integral equations of Volterra type.

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