

Dedicated to Professor Emeritus Ioan A. Rus on the occasion of his 80th anniversary

A study of a system of operator inclusions via a fixed point approach and applications to functional-differential inclusions

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ABSTRACT.

In this paper, some existence results for a system of operator inclusions are presented. Qualitative properties of the solution set are also discussed. The method is based on the application of a fixed point theorem for an appropriate operator on the Cartesian product of the given spaces. The approach is new even for the case of the metric spaces. As an application, an existence result for a mixed boundary and initial value problem for a system of second order differential inclusions is given.

Acknowledgement. For the first author, this paper was supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-ID-PCE-2011-3-0094.

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Received: 24.05.2016; In revised form: 04.07.2016; Accepted: 10.07.2016

2010 Mathematics Subject Classification. 47H10, 34B15, 54H25.

Key words and phrases. Multi-valued operator, fixed point, ordered metric space, coupled fixed point, data dependence, well-posedness, Ulam-Hyers stability, limit shadowing, measurable selection, integral inclusion, boundary value problem, initial value problem.

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