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Iterative methods for generalized split feasibility problems in Banach spaces

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Abstract.

Inspired by the recent work of Takahashi et al. [W. Takahashi, H.-K. Xu and J.-C. Yao, *Iterative methods for generalized split feasibility problems in Hilbert spaces*, Set-Valued Var. Anal., **23** (2015), 205–221], in this paper, we study generalized split feasibility problems (GSFPs) in the setting of Banach spaces. We propose iterative algorithms to compute the approximate solutions of such problems. The weak convergence of the sequence generated by the proposed algorithms is studied. As applications, we derive some algorithms and convergence results for some problems from nonlinear analysis, namely, split feasibility problems, equilibrium problems, etc. Our results generalize several known results in the literature including the results of Takahashi et al. [W. Takahashi, H.-K. Xu and J.-C. Yao, *Iterative methods for generalized split feasibility problems in Hilbert spaces*, Set-Valued Var. Anal., **23** (2015), 205–221].

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