

Extending the applicability of Newton's method using Wang's– Smale's α -theory

IOANNIS K. ARGYROS and SANTHOSH GEORGE

ABSTRACT.

We improve semilocal convergence results for Newton's method by defining a more precise domain where the Newton iterate lies than in earlier studies using the Smale's α - theory. These improvements are obtained under the same computational cost. Numerical examples are also presented in this study to show that the earlier results cannot apply but the new results can apply to solve equations.

REFERENCES

- [1] Amat, S., Busquier, S. and Negra, M., *Adaptive approximation of nonlinear operators*, Numer. Funct. Anal. Optim., **25** (2004), 397–405
- [2] Argyros, I. K. and Hilout, S., *Weaker conditions for the convergence of Newton's method*, J. Complexity, **28** (2012), 364–387
- [3] Argyros, I. K. and Hilout, S., *Computational methods in nonlinear analysis, Efficient Algorithms, Fixed point Theory and Applications*, World Acientific, Singapore, 2013
- [4] Argyros, I. K., Hilout, S. and Khattri, S. K., *Expanding the applicability of Newton's method using Smale's α -theory*, J. Comput. Appl. Math., **261** (2014) 183–200
- [5] Cienciaruso, F., *Convergence of Newton-Kantorovich approximations to an approximate zero*, Numer. Funct. Anal. Optim., **28** (2007) 631–645
- [6] Dedieu, J. P., *Points fixes, zéros et la méthode de Newton*, **54**, Springer, Berlin, 2006
- [7] Ezquerro, J. A., Gutiérrez, J. M., Hernández, M. A., Romero, N. and Rubio , M. J, *The Newton method: from Newton to Kantorovich*, (Spanish) Gac. R. Soc. Mat. Esp., **13** (2010) 53–76
- [8] Proinov, P. D., *New general convergence theory for iterative processes and its applications to Newton-Kantorovich type theorems*, J. Complexity, **26** (2010) 3–42
- [9] Smale, S., *Newton's method estimates from data at one point. The merging of disciplines: new directions in pure, applied and computational mathematics* (Laramie, Wyo., 1985), 185–196, Springer, New York, 1986
- [10] Wang, X. H., *Convergence of Newton's method and inverse function theorem in Banach space*, Math. Comp., **68** (1999) 169–186

DEPARTMENT OF MATHEMATICAL SCIENCES
CAMERON UNIVERSITY
LAWTON, OK 73505, USA
E-mail address: iargyros@cameron.edu

DEPARTMENT OF MATHEMATICAL AND COMPUTATIONAL SCIENCES
NIT KARNATAKA, INDIA-575 025
E-mail address: sgeorge@nitk.ac.in

Received: 27.10.2015; In revised form: 24.08.2016; Accepted: 29.08.2016

2010 Mathematics Subject Classification. 65D10, 65D99.

Key words and phrases. Newton's method, Banach space, semi-local convergence, Smale's α - theory, Fréchet-derivative.

Corresponding author: Ioannis K. Argyros; iargyros@cameron.edu