Quantitative estimates in uniform and pointwise approximation by Bernstein-Durrmeyer-Choquet operators

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ABSTRACT.

For the qualitative results of uniform and pointwise approximation obtained in [8], we present here general quantitative estimates in terms of the modulus of continuity and of a K-functional, in approximation by the generalized multivariate Bernstein-Durrmeyer operator $M_{n,\Gamma_{n,x}}$, written in terms of Choquet integrals with respect to a family of monotone and submodular set functions, $\Gamma_{n,x}$, on the standard d-dimensional simplex. If $d = 1$ and the Choquet integrals are taken with respect to some concrete possibility measures, the estimate in terms of the modulus of continuity is detailed. Examples improving the estimates given by the classical operators also are presented.

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