

Bézier variant of genuine-Durrmeyer type operators based on Pólya distribution

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ABSTRACT.

In this paper we introduce the Bézier variant of genuine-Durrmeyer type operators having Pólya basis functions. We give a global approximation theorem in terms of second order modulus of continuity, a direct approximation theorem by means of the Ditzian-Totik modulus of smoothness and a Voronovskaja type theorem by using the Ditzian-Totik modulus of smoothness. The rate of convergence for functions whose derivatives are of bounded variation is obtained. Further, we show the rate of convergence of these operators to certain functions by illustrative graphics using the Maple algorithms.

Acknowledgement. The first author is thankful to the "Ministry of Human Resource Development", India for financial support to carry out the above research work. The work of the second author was financed from Lucian Blaga University of Sibiu research grants LBUS-IRG-2015-01, No.2032/7.

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Received: 29.10.2015; In revised form: 08.01.2016; Accepted: 26.03.2016

2010 Mathematics Subject Classification. 41A25, 26A15.

Key words and phrases. Bézier operators, genuine-Durrmeyer type operators, Pólya distribution, rate of convergence, bounded variation.

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