

Hardy-Littlewood-Pólya theorem of majorization in the framework of generalized convexity

CONSTANTIN P. NICULESCU^{1,2} and IONEL ROVENȚA¹

ABSTRACT.

Based on a new concept of generalized relative convexity, a large extension of Hardy-Littlewood-Pólya theorem of majorization is obtained. Several applications escaping the classical framework of convexity are included.

Acknowledgement. The first author was supported by Romanian National Authority for Scientific Research CNCS – UEFISCDI research project PN-II-ID-PCE-2011-3-0257 and the second author was supported by Romanian National Authority for Scientific Research CNCS – UEFISCDI research project PN-II-RU-TE-2014-4-1109.

REFERENCES

- [1] Anderson, G. D., Vamanamurthy, M. K. and Vuorinen, M., *Generalized convexity and inequalities*, J. Math. Anal. Appl., **335** (2007), 1294–1308
- [2] Aumann, G., *Konvexe Funktionen und Induktion bei Ungleichungen zwischen Mittelwerten*, Bayer. Akad. Wiss. Math.-Natur. Kl. Abh., Math. Ann., **109** (1933), 405–413
- [3] Borwein, D., Borwein, J., Fee, G. and Girgensohn, R., *Refined convexity and special cases of the Blaschke-Santaló inequality*, Math. Inequal. Appl., **4** (2001), 631–638
- [4] Czinder, P. and Páles, Z., *An extension of the Hermite-Hadamard inequality and an application for Gini and Stolarsky means*, JIPAM. J. Inequal. Pure Appl. Math., **5** (2004), No. 2, Article 42, 8 pp. (electronic)
- [5] Dragomirescu, M. and Ivan, C., *Convexity-like inequalities for averages in a convex set*, Aequationes Math., **45** (1993), 179–194
- [6] Florea, A. and Niculescu, C. P., *A Hermite-Hadamard inequality for convex-concave symmetric functions*, Bull. Math. Soc. Sci. Math. Roumanie (N.S.), **50 (98)** (2007), No. 2, 149–156
- [7] Lindenstrauss, J. and Tzafriri, L., *Classical Banach spaces*, **2**, Springer-Verlag, 1979
- [8] Tung-Po Lin, *The Power Mean and the Logarithmic Mean*, Am. Math. Mon., **81** (1974), No. 8, 879–883
- [9] Marshall, A. W., Olkin, I. and Arnold, B., *Inequalities: Theory of Majorization and Its Application*, 2nd Edition, Springer, 2011
- [10] Mihai, M. and Niculescu, C. P., *A simple proof of the Jensen type inequality of Fink and Jodeit*, Mediterr. J. Math., **13** (2016), No. 1, 119–126
- [11] Neuman, E., *The weighted logarithmic mean*, J. Math. Anal. Appl., **188** (1994), 885–900
- [12] Niculescu, C. P., *The Hermite-Hadamard inequality for log-convex functions*, Nonlinear Analysis, **75** (2012), 662–669
- [13] Niculescu, C. P. and Persson, L.-E., *Convex Functions and their Applications. A Contemporary Approach*, CMS Books in Mathematics, **23**, Springer-Verlag, New York, 2006
- [14] Niculescu, C. P. and Roventă, I., *An Approach of Majorization in Spaces with a Curved Geometry*, J. Math. Anal. Appl., **411** (2014), No. 1, 119–128
- [15] Niculescu, C. P. and Roventă, I., *Relative convexity and its applications*, Aequationes Math., **89** (2015), No. 5, 1389–1400
- [16] Niculescu, C. P. and Spiridon, C., *New Jensen-type inequalities*, J. Math. Anal. Appl., **401** (2013), No. 1, 343–348

Received: 26.12.2015; In revised form: 29.06.2016; Accepted: 20.07.2016

2010 Mathematics Subject Classification. 26B25, 26D15.

Key words and phrases. Convex function, supporting hyperplane, majorization, mean.

Corresponding author: Ionel Roventă; ionelroventa@yahoo.com

- [17] Niculescu, C. P. and Stephan, H., *Lagrange's Barycentric Identity From An Analytic Viewpoint*, Bull. Math. Soc. Sci. Math. Roumanie, **56** (104) (2013), 487–496
- [18] Pearce, C. E. M. and Pečarić, J., *On convexity-like inequalities*, Revue Roumaine Math. Pure Appl. Math., **42** (1997), No. 1-2, 133–138
- [19] Pečarić, J., Proschan, F. and Tong, Y. L., *Convex functions, partial orderings, and statistical applications*, Academic Press, 1992
- [20] Sandor, J. and Trif, T., *A new refinement of the Ky Fan inequality*, Math. Inequal. Appl., **2** (1999), 529–533
- [21] Simon, B., *Convexity: An Analytic Viewpoint*, Cambridge University Press, 2011

¹DEPARTMENT OF MATHEMATICS
UNIVERSITY OF CRAIOVA
A. I. CUZA 13, 200585 CRAIOVA, ROMANIA
E-mail address: cpniculescu@gmail.com
E-mail address: ionelroventa@yahoo.com

²THE ACADEMY OF ROMANIAN SCIENTISTS
SPLAIUL INDEPENDENȚEI 54, BUCHAREST RO-050094, ROMANIA
E-mail address: cpniculescu@gmail.com