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Blow up of solutions for 3D quasi-linear wave equations with positive initial energy

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ABSTRACT.

In this paper we investigate blow up property of solutions for a system of nonlinear wave equations with nonlinear dissipations and positive initial energy in a bounded domain in $\mathbb{R}^3$. Our result improves and extends earlier results in the literature such as the ones in [Zhou, J. and Mu, C., *The lifespan for 3D quasilinear wave equations with nonlinear damping terms*, Nonlinear Anal., **74** (2011), 5455–5466] and [Pişkin, E., *Uniform decay and blow-up of solutions for coupled nonlinear Klein-Gordon equations with nonlinear damping terms*, Math. Meth. Appl. Scie., **37** (2014), No. 18, 3036–3047] in which the nonexistence results obtained only for negative initial energy or the one in [Ye, Y., *Global existence and nonexistence of solutions for coupled nonlinear wave equations with damping and source terms*, Bull. Korean Math. Soc., **51** (2014), No. 6, 1697–1710] where blow up results have been not addressed. Estimate for the lower bound of the blow up time is also given.

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