

Normality degrees of finite groups

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ABSTRACT.

In this paper we introduce and study the concept of normality degree of a finite group G . This quantity measures the probability of a random subgroup of G to be normal. Explicit formulas are obtained for some particular classes of finite groups. Several limits of normality degrees are also computed.

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