

Isosceles triple convexity

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ABSTRACT.

A set S in \mathbb{R}^d is called *it-convex* if, for any two distinct points in S , there exists a third point in S , such that one of the three points is equidistant from the others.

In this paper we first investigate nondiscrete *it-convex* sets, then discuss about the *it-convexity* of the eleven Archimedean tilings, and treat subsequently finite subsets of the square lattice. Finally, we obtain a lower bound on the number of isosceles triples contained in an n -point *it-convex* set.

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