Tzitzeica equations and Tzitzeica surfaces in separable coordinate systems and the Ricci flow tensor field

WLADIMIR G. BOSKOFF, MIRCEA CRASMAREANU and LAURIAN-IOAN PIŞCORAN

ABSTRACT.

The Tzitzeica equation and two well-known Tzitzeica surfaces are studied in the separable coordinate systems on the plane and space respectively. We study also Tzitzeica graphs with a parameter and interpret the induced class of first fundamental forms as a Riemannian flow. Consequently, we introduce a tensor field which measures how far is a given Riemannian flow to be a Ricci one. This tensor field is explicitly computed for the case of a initial isothermic metric and a flow of convex type.

REFERENCES

[1] Agnew, A. F., Bobe, A., Boskoff, W. G. and Suceava, B. D., *Gheorghe Ţiţeica and the origins of affine differential geometry*, Historia Math., **36** (2009), No. 2, 161–170, MR2509052

[2] Agnew, A. F., Bobe, A., Boskoff, W. G., Homentcovschi, L. and Suceava, B. D.,*The equation of Euler's line yields a Tzitzeica surface*, Elem. Math., **64** (2009), No. 2, 71–80, MR2495804 (2010d:51028)

[3] Babalic C. N., Constantinescu, R. and Gerdjikov, V. S., *On the soliton solutions of a family of Tzitzeica equations*, J. Geom. Symmetry Phys, **37** (2015), 1–24, MR3362493

[4] Bîla, N., *Symmetry groups and Lagrangians associated with Tzitzeica surfaces*, Balkan J. Geom. Appl., **10** (2005), No. 1, 73–91, MR2209917 (2006k:53012)

[5] Bobe A., Boskoff, W. G. and Ciuca, M. G, *Tzitzeica-type centro-affine invariants in Minkowski spaces*, An. Stiint. Univ. Ovidius Constanta Ser. Mat., **20** (2012), No. 2, 27–34, MR2945953

[6] Callahan, J. J. *The geometry of spacetime. An introduction to special and general relativity*, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 2000, MR1731438 (2001i:83001)

[7] Chow, B., Lu, P. and Ni, L., *Hamilton's Ricci flow*, Graduate Studies in Mathematics, 77, American Mathematical Society, Providence, RI; Science Press, New York (2006), MR2274812 (2008a:53068)

[8] Constantinescu, O. and Crasmareanu, M., *A new Tzitzeica hypersurface and cubic Finslerian metrics of Berwald type*, Balkan J. Geom. Appl., **16** (2011), No. 2, 27–34, MR2785729 (2012e:53032)

[9] Crasmareanu, M., *Tzitzeica indicatrices in Lagrange geometry*, Algebras Groups Geom., **21** (2004), No. 3, 277–284, MR2117022 (2005k:53017)

[10] Crasmareanu, M., *Tzitzeica figuratrices in Hamilton geometry*, Balkan J. Geom. Appl., **10** (2005), No. 1, 92–97, MR2209918 (2007d:53030)

[11] Grinfeld, P., *Introduction to tensor analysis and the calculus of moving surfaces*, New York, Springer, 2013, MR3136419

[12] Kalnins, E. G., Williams, G. C., Miller Jr., W. and Pogosyan, G. S., *Superintegrability in three-dimensional Euclidean space*, J. Math. Phys., **40** (1999), No. 2, 708–725, MR1674226 (99m:81044)

[13] Krivoshapko, S. N. and Ivanov, V. N., *Encyclopedia of analytical surfaces*, Springer, Cham, 2015, MR3309742

[14] Miller Jr., W., *Multiseparability and superintegrability for classical and quantum systems*, in Integrable systems: from classical to quantum (Montréal, QC, 1999), 129–156, CRM Proc. Lecture Notes, 26, Amer. Math. Soc., Providence, RI, 2000, MR1791888 (2001h:37117)

[15] Rogers, C. and Schief, W. K., *Bäcklund and Darboux transformations. Geometry and modern applications in soliton theory*, Cambridge Texts in Applied Mathematics, Cambridge University Press, Cambridge, 2002, MR1908706 (2003j:37120)

[16] Rui, W., *Exact traveling wave solutions for a nonlinear evolution equation of generalized Tzitzica-Dodd-Bullough-Mikhailov type*, J. Appl. Math., 2013, Art. ID 395628, 14 pp. MR3064955

[17] Tzitzeica, G., *Sur une Nouvelle Classe de Surfaces*, C. R. Acad. Sci. Paris, **144** (1907), 1257–1259, JFM 38.0642.01

[18] Țurcanu, T. and Udriște, C., *Tzitzeica geometry of soliton solutions for quartic interaction PDE*, Balkan J. Geom. Appl., **21** (2016), No. 1, 103–112, MR3511141

[19] Udriste, C., Arsinte, V. and Cipu, C., *Tzitzeica and sine-Gordon solitons*, Balkan J. Geom. Appl., **16** (2011), No. 1, 150–154, MR2785723 (2012e:37150)

[20] Zeng, W. and Gu, X. D., *Ricci flow for shape analysis and surface registration. Theories, algorithms and applications*, Springer Briefs in Mathematics, Springer, New York, 2013, MR3136003

DEPARTMENT OF MATHEMATICS
OVIDIUS UNIVERSITY OF CONSTANTA
900527 CONSTANTA, ROMANIA
E-mail address: `boskoff@univ-ovidius.ro`

FACULTY OF MATHEMATICS
UNIVERSITY "AL. I. CUZA"
700506 IAȘI, ROMANIA
E-mail address: `mcrasm@uaic.ro`

NORTH UNIVERSITY CENTER OF BAIA MARE
TECHNICAL UNIVERSITY OF CLUJ NAPOCA
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
VICTORIEI 76, 430122 BAIA MARE, ROMANIA
E-mail address: `plaurian@yahoo.com`