

Köthe-Bochner spaces that are Hilbert spaces

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ABSTRACT.

We are concerned with Köthe-Bochner spaces that are Hilbert spaces (resp. hilbertable spaces). It is shown that this is equivalent to the fact that, separately, L_ρ and X are Hilbert spaces (resp. hilbertable spaces). The complete characterization of the L_ρ spaces that are Hilbert spaces, given by the first-author, is used.

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