

## Two open problems in the fixed point theory of contractive type mappings on quasimetric spaces

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### ABSTRACT.

Two open problems in the fixed point theory of quasi metric spaces posed in [Berinde, V. and Choban, M. M., *Generalized distances and their associate metrics. Impact on fixed point theory*, Creat. Math. Inform., **22** (2013), No. 1, 23–32] are considered. We give a complete answer to the first problem, a partial answer to the second one, and also illustrate the complexity and relevance of these problems by means of four very interesting and comprehensive examples.

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