Existence of solutions for Caputo fractional boundary value problems with integral conditions

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ABSTRACT.

In this paper, we investigate the existence results for Caputo fractional boundary value problems with integral conditions. Our analysis relies on Banach's contraction principle, Leray-Schauder nonlinear alternative, Boyed and Wong fixed point theorem, and Krasnoselskii's fixed point theorem. As applications, some examples are provided to illustrate our main results.

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