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Abstract linear second order differential equations with two small parameters and depending on time operators

ANDREI PERJAN and GALINA RUSU

ABSTRACT.

In a real Hilbert space H consider the following singularly perturbed Cauchy problem

$$\begin{cases} \varepsilon\, u''_{\varepsilon\delta}(t) + \delta\, u'_{\varepsilon\delta}(t) + A(t)u_{\varepsilon\delta}(t) = f(t), \quad t \in (0,T), \\ u_{\varepsilon\delta}(0) = u_0, \quad u'_{\varepsilon\delta}(0) = u_1, \end{cases}$$

where $A(t) : V \subset H \to H$, $t \in [0,\infty)$, is a family of linear self-adjoint operators, $u_0, u_1 \in H$, $f : [0,T] \mapsto H$ and ε, δ are two small parameters.

We study the behavior of solutions $u_{\varepsilon\delta}$ to this problem in two different cases: $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$; $\varepsilon \to 0$ and $\delta \to 0$, relative to solution to the corresponding unperturbed problem.

We obtain some *a priori* estimates of solutions to the perturbed problem, which are uniform with respect to parameters, and a relationship between solutions to both problems. We establish that the solution to the perturbed problem has a singular behavior, relative to the parameters, in the neighbourhood of $t = 0$. We show the boundary layer and boundary layer function in both cases.

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Corresponding author: Rusu G.; rusugalinamoldova@gmail.com

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Department of Mathematics
Moldova State University
A. Mateevici 60, MD 2009 Chisinau, Republic of Moldova
E-mail address: aperjan1948@gmail.com
E-mail address: rusugalinamoldova@gmail.com