

Walk-set induced connectedness in digital spaces

JOSEF ŠLAPAL

ABSTRACT.

In an undirected simple graph, we define connectedness induced by a set of walks of the same lengths. We show that the connectedness is preserved by the strong product of graphs with walk sets. This result is used to introduce a graph on the vertex set \mathbb{Z}^2 with sets of walks that is obtained as the strong product of a pair of copies of a graph on the vertex set \mathbb{Z} with certain walk sets. It is proved that each of the walk sets in the graph introduced induces connectedness on \mathbb{Z}^2 that satisfies a digital analogue of the Jordan curve theorem. It follows that the graph with any of the walk sets provides a convenient structure on the digital plane \mathbb{Z}^2 for the study of digital images.

Acknowledgement. This work was supported by the Ministry of Education, Youth and Sports of the Czech Republic from the National Programme of Sustainability (NPU II) project "IT4Innovations excellence in science - LQ1602".

REFERENCES

- [1] Harrary, F., *Graph Theory*, Addison-Wesley Publ. Comp., Reading, Massachusetts, Menlo Park, California, London, Don Mills, Ontario, 1969
- [2] Khalimsky, E. D, Kopperman, R. and Meyer, P. R., *Computer graphics and connected topologies on finite ordered sets*, *Topology Appl.*, **36** (1990), 1–17
- [3] Kong, T. Y. and Roscoe, W., *A theory of binary digital pictures*, *Comput. Vision Graphics Image Process.*, **32** (1985), 221–243
- [4] Kong, T. Y. and Rosenfeld, A., *Digital topology: Introduction and survey*, *Comput. Vision Graphics Image Process.*, **48** (1989), 357–393
- [5] Kong, T. Y, Kopperman, R. and Meyer, P. R., *A topological approach to digital topology*, *Amer. Math. Monthly*, **98** (1991), 902–917
- [6] Rosenfeld, A., *Connectivity in digital pictures*, *J. Assoc. Comput. Mach.*, **17** (1970), 146–160
- [7] Rosenfeld, A., *Digital topology*, *Amer. Math. Monthly*, **86** (1979), 621–630
- [8] Sabidussi, G., *Graph multiplication*, *Math. Z.*, **72** (1960), 446–457
- [9] Šlapal, J., *Jordan curve theorems with respect to certain pretopologies on \mathbb{Z}^2* , *Lect. Notes Comput. Sci.*, **5810** (2009), 252–262
- [10] Šlapal, J., *A Jordan curve theorem in the digital plane*, *Lect. Notes Comput. Sci.*, **6636** (2011), 120–131
- [11] Šlapal, J., *Topological structuring of the digital plane*, *Discr. Math. Theoret. Comput. Sci.*, **15** (2013), 165–176
- [12] Šlapal, J., *Graphs with a walk partition for structuring digital spaces*, *Inf. Sciences* **233** (2013), 305–312

BRNO UNIVERSITY OF TECHNOLOGY
IT4INNOVATIONS CENTRE OF EXCELLENCE
TECHNICKÁ 2, 616 69 BRNO, CZECH REPUBLIC
E-mail address: slapal@fme.vutbr.cz

Received: 23.11.2016; In revised form: 04.05.2017; Accepted: 11.05.2017

2010 *Mathematics Subject Classification.* 05C40, 68R10.

Key words and phrases. *Simple graph, strong product, walk, connectedness, digital space, Jordan curve theorem.*