

A general concept of multiple fixed point for mappings defined on spaces with a distance

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ABSTRACT.

Our main aim in this paper is to introduce a general concept of multidimensional fixed point of a mapping in spaces with distance and establish various multidimensional fixed point results. This new concept simplifies the similar notion from [A. Roldan, J. Martinez-Moreno, C. Roldan, *Multidimensional fixed point theorems in partially ordered complete metric spaces*, J. Math. Anal. Appl. 396 (2012), 536–545]. The obtained multiple fixed point theorems extend, generalise and unify many related results in literature.

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