

Approximating common fixed point of asymptotically nonexpansive mappings without convergence condition

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ABSTRACT.

In the context of a hyperbolic space, we introduce and study convergence of an implicit iterative scheme of a finite family of asymptotically nonexpansive mappings without convergence condition. The results presented substantially improve and extend several well-known results in uniformly convex Banach spaces.

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