

Goebel and Kirk fixed point theorem for multivalued asymptotically nonexpansive mappings

M. A. KHAMSI^{1,2} and A. R. KHAN²

ABSTRACT.

We introduce the concept of a multivalued asymptotically nonexpansive mapping and establish Goebel and Kirk fixed point theorem for these mappings in uniformly hyperbolic metric spaces. We also define a modified Mann iteration process for this class of mappings and obtain an extension of some well-known results for singlevalued mappings defined on linear as well as nonlinear domains.

Acknowledgement. The authors acknowledge gratefully the support of KACST, Riyadh, Saudi Arabia for supporting Research Project ARP-32-34.

REFERENCES

- [1] Alber, Ya. I., Chidume, C. E. and Zegeye, H., *Approximating fixed points of total asymptotically nonexpansive mappings*, Fixed Point Theory Appl., 2006, Art. ID 10673, 20 pp.
- [2] Bridson, M. and Haefliger, A., *Metric Spaces of Non-positive Curvature*, Springer-Verlag, Berlin, 1999
- [3] Busemann, H., *Spaces with non-positive curvature*, Acta. Math., **80** (1948), 259–310
- [4] Chang, S. S., Wang, L., Lee, H. W. J., Chan, C. K. and Yang, L., *Demiclosed principle and Δ -convergence theorems for total asymptotically nonexpansive mappings in CAT(0) spaces*, Appl. Math. Comput., **219** (2012), No. 5, 2611–2617
- [5] Dhompongsa, S., Kirk, W. A. and Panyanak, B., *Nonexpansive set-valued mappings in metric and Banach spaces*, J. Nonlinear Convex Anal., **8** (2007), 35–45
- [6] Fukhar-Ud-Din, H., *Existence and approximation of fixed points in convex metric spaces*, Carpathian J. Math., **30** (2014), 175–185
- [7] Goebel, K. and Kirk, W. A., *A fixed point theorem for asymptotically nonexpansive mappings*, Proc. Amer. Math. Soc., **35** (1972), 171–174
- [8] Goebel, K. and Kirk, W. A., *Topics in Metric fixed point theory*, Cambridge University Press, Cambridge, 1990
- [9] Goebel, K. and Reich, S., *Uniform Convexity, Hyperbolic Geometry, and Nonexpansive Mappings*, Series of Monographs and Textbooks in Pure and Applied Mathematics, Vol. **83**, Dekker, New York, 1984
- [10] Górniewicz, L., *Topological Fixed Point Theory of Multivalued Mappings*, Mathematics and its Applications, vol. **495**, Kluwer Academic Publishers, Dordrecht, 1999
- [11] Ibn Dehaish, B., Khamsi, M. A. and Khan, A. R., *Mann iteration process for asymptotic pointwise nonexpansive mappings in metric spaces*, J. Math. Anal. Appl., **397** (2013), 861–868
- [12] Khamsi, M. A. and Kirk, W. A., *On Uniformly Lipschitzian Multivalued Mappings in Banach and Metric spaces*, Nonlinear Anal., **72** (2010), 2080–2085
- [13] Khamsi, M. A. and Kirk, W. A., *An Introduction to Metric Spaces and Fixed Point Theory*, Pure and Applied Math., Wiley, New York 2001
- [14] Khamsi, M. A., *On metric spaces with uniform normal structure*, Proc. Am. math. Soc., **106** (1989), 723–726
- [15] Khamsi, M. A. and Khan, A. R., *Inequalities in Metric Spaces with Applications*, Nonlinear Anal., **74** (2011), 4036–4045
- [16] Khan, A. R., *Properties of fixed point set of a multivalued map*, J. Appl. Math. Stochas. Anal., 2005:3 (2005), 323–331

Received: 19.12.2016; In revised form: 04.05.2017; Accepted: 15.05.2017

2010 Mathematics Subject Classification. 47H10, 47E10.

Key words and phrases. Asymptotically nonexpansive mapping, fixed point, generalized orbit, Mann iteration process, multivalued mapping.

Corresponding author: A. R. Khan; arahim@kfupm.edu.sa

- [17] Kirk, W. A., *Fixed point theory for nonexpansive mappings, I and II*, Lecture Notes in Mathematics, Springer, Berlin, **886** (1981), 485–505
- [18] Kirk, W. A., *Geodesic geometry and fixed point theory II*, in: J. Garcia Falset, E. Llorens Fuster, B. Sims (Eds.), *Fixed Point Theory and Applications*, Yokohama Publ., (2004) 113–142
- [19] Kirk, W. A., *Fixed point theorems in $CAT(0)$ spaces and R -trees*, *Fixed Point Theory Appl.*, **4** (2004), 309–316
- [20] Leustean, L., *A quadratic rate of asymptotic regularity for $CAT(0)$ -spaces*, *J. Math. Anal. Appl.*, **325** (2007), 386–399
- [21] Lin, P. K., Tan, K. K. and Xu, H. K., *Demiclosedness principle and asymptotic behavior for asymptotically nonexpansive mappings*, *Nonlinear Anal.*, **24** (1995), 929–946
- [22] Menger, K., *Untersuchungen über allgemeine Metrik*, *Math. Ann.*, **100** (1928), 75–163
- [23] Nanjaras, B. and Panyanak, B., *Demiclosed principle for asymptotically nonexpansive mappings in $CAT(0)$ -spaces*, *Fixed Point Theory Appl.*, 2010, Art. ID 268780, 14 pp.
- [24] Pansuwan, A. and Sintunavarat, W., *A new iterative scheme for numerical reckoning fixed points of total asymptotically nonexpansive mappings*, *Fixed Point Theory Appl.*, 2016, 2016:83, 13 pp.
- [25] Panyanak, B., *On total asymptotically nonexpansive mappings in $CAT(\kappa)$ spaces*, *J. Inequal. Appl.*, 2014, 2014:336, 13 pp.
- [26] Petrusel, A., *Multivalued weakly Picard operators and applications*, *Sci. Math. Japan*, **59** (2004), 169–202
- [27] Reich, S. and Shafrir, I., *Nonexpansive iterations in hyperbolic spaces*, *Nonlinear Anal.*, **15** (1990), 537–558
- [28] Rus, I. A., *Basic problems of the metric fixed point theory revisited*, II. *Studia Univ. Babeş-Bolyai Math.*, **36** (1991), 81–99
- [29] Rus, I. A., Petrusel, A. and Sintamarian, A., *Data dependence of the fixed points set of multivalued weakly Picard operators*, *Studia Univ. Babeş-Bolyai Math.*, **46** (2001), No. 2, 111–121
- [30] Rus, I. A., Petrusel, A. and Sintamarian, A., *Data dependence of the fixed points set of some multivalued weakly Picard operators*, *Nonlinear Anal.*, **52** (2003), 1947–1959
- [31] Schu, J., *Weak and strong convergence to fixed points of asymptotically nonexpansive mappings*, *Bull. Austral. Math. Soc.*, **43** (1991), 153–159
- [32] Zhang, S. S., Wang, L., Zhao, Y. H. and Wang, G., *Strong convergence of multivalued Bregman totally quasi-asymptotically nonexpansive mappings*, (Chinese) *Acta Math. Sinica (Chin. Ser.)*, **58** (2015), No. 2, 213–226

¹DEPARTMENT OF MATHEMATICAL SCIENCES
 THE UNIVERSITY OF TEXAS AT EL PASO
 EL PASO, TX 79968, U.S.A.
 E-mail address: mohamed@utep.edu

² DEPARTMENT OF MATHEMATICS & STATISTICS
 KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
 DHAHRAN 31261, SAUDI ARABIA
 E-mail address: mohamed@utep.edu
 E-mail address: arahim@kfupm.edu.sa