CARPATHIAN J. MATH. Online version available at http://carpathian.ubm.ro
33 (2017), No. 3, 389-397 Print Edition: ISSN 1584 - 2851 Online Edition: ISSN 1843 - 4401

Projection algorithms for composite minimization

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ABSTRACT.

Parallel and cyclic projection algorithms are proposed for minimizing the sum of a finite family of convex functions over the intersection of a finite family of closed convex subsets of a Hilbert space. These algorithms consist of two steps. Once the kth iterate is constructed, an inner circle of gradient descent process is executed through each local function, and then a parallel or cyclic projection process is applied to produce the $(k+1)$ iterate. These algorithms are proved to converge to an optimal solution of the composite minimization problem under investigation upon assuming boundedness of the gradients at the iterates of the local functions and the stepsizes being chosen appropriately.

Acknowledgement. The authors were grateful to the reviewers for their careful reading and helpful comments which improved the presentation of this manuscript.

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Received: 28.11.2016; In revised form: 02.03.2017; Accepted: 22.03.2017
2010 *Mathematics Subject Classification.* 90C25, 90C52, 65K10, 47J25.
Key words and phrases. *convex feasibility, composite minimization, projection algorithm.*
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