

Bounds for some entropies and special functions

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ABSTRACT.

We consider a family of probability distributions depending on a real parameter and including the binomial, Poisson and negative binomial distributions. The corresponding index of coincidence satisfies a Heun differential equation and is a logarithmically convex function. Combining these facts we get bounds for the index of coincidence, and consequently for Rényi and Tsallis entropies of order 2.

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