

# Painlevé-Kuratowski convergences of the approximate solution sets for vector quasiequilibrium problems

NGUYEN VAN HUNG<sup>1</sup>, DINH HUY HOANG<sup>2</sup> and VO MINH TAM<sup>1</sup>

## ABSTRACT.

In this paper, we study vector quasiequilibrium problems. After that, the Painlevé-Kuratowski upper convergence, lower convergence and convergence of the approximate solution sets for these problems are investigated by using a sequence of mappings  $\Gamma_C$ -converging. As applications, we also consider the Painlevé-Kuratowski upper convergence of the approximate solution sets in the special cases of variational inequality problems of the Minty type and Stampacchia type. The results presented in this paper extend and improve some main results in the literature.

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Corresponding author: Nguyen Van Hung; [ngvhungdhd@yahoo.com](mailto:ngvhungdhd@yahoo.com)

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<sup>1</sup>DEPARTMENT OF MATHEMATICS  
DONG THAP UNIVERSITY  
DONG THAP, VIETNAM  
E-mail address: ngvhungdhdt@yahoo.com  
E-mail address: vmtam@dt hu.edu.vn

<sup>2</sup> DEPARTMENT OF MATHEMATICS  
VINH UNIVERSITY  
NGHE AN, VIETNAM  
E-mail address: dhhoangdhv@gmail.com