

# On some parameters in the space of regulated functions and their applications

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## ABSTRACT.

In this paper, we study a class of discontinuous functions being a space of solutions for some differential and integral equations. We investigate functions having finite one-sided limits, i.e. regulated functions. In the space of such functions, we introduce some new concepts like a modulus of equi-regularity or a measure of noncompactness, allowing us to unify the proofs for the results about existence for both continuous and discontinuous solutions. An example of applications for quadratic integral equations, essentially improving earlier ones, completes the paper.

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