

Existence of a unique positive solution for a singular fractional boundary value problem

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ABSTRACT.

In the present work, we discuss the existence of a unique positive solution of a boundary value problem for a nonlinear fractional order equation with singularity. Precisely, order of equation $D_{0+}^{\alpha} u(t) = f(t, u(t))$ belongs to $(3, 4]$ and f has a singularity at $t = 0$ and as a boundary conditions we use $u(0) = u(1) = u'(0) = u'(1) = 0$. Using a fixed point theorem, we prove the existence of unique positive solution of the considered problem.

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