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Best proximity point theorems for G-proximal weak contractions in complete metric spaces endowed with graphs

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ABSTRACT.

In this paper, the existence of best proximity point theorems for two new types of nonlinear non-self mappings in a complete metric space endowed with a directed graph are established. Our main results extend and generalize many known results in the literatures. As a special case of the main results, the best proximity point theorems on partially ordered sets are obtained.

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