Multiple existence of solutions for a coupled system involving the distributional Henstock-Kurzweil integral

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ABSTRACT.

This paper deals with a coupled system in the sense of distributions (generalized functions). Our main goal is to get the basic multiple existence results via some degree theory arguments. Differently from the literatures, the proof is based on the concept of a general integral named distributional Henstock-Kurzweil integral, which includes the Lebesgue and Henstock-Kurzweil integrals as special cases. Finally, an example is given to illustrate that the presented abstract theory contains some previous results as special cases.

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