

Cyclic permutations and crossing numbers of join products of symmetric graph of order six

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ABSTRACT.

In the paper, we extend known results concerning crossing numbers for join of graphs of order six. We give the crossing number of the join product $G + D_n$, where the graph G consists of two leaves incident with two opposite vertices of one 4-cycle, and D_n consists on n isolated vertices. The proof is done with the help of software that generates all cyclic permutations for a given number k , and creates a new graph COG for a calculating the distances between all $(k - 1)!$ vertices of the graph. Finally, by adding new edges to the graph G , we are able to obtain the crossing number of the join product with the discrete graph D_n for two other graphs. The methods used in the paper are new, and they are based on combinatorial properties of cyclic permutations.

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