

# Multiple positive solutions to a $(2m)$ th-order boundary value problem

HABIBA BOULAIKI, TOUFIK MOUSSAOUI and RADU PRECUP

## ABSTRACT.

The aim of the present paper is to study the existence, localization and multiplicity of positive solutions for a  $(2m)$ th-order boundary value problem subject to the Dirichlet conditions. Our approach is based on critical point theory in conical shells and Harnack type inequalities.

## REFERENCES

- [1] Avery, R. I. and Henderson, J., *Two positive fixed points of nonlinear operators on ordered Banach spaces*, Comm. Appl. Nonlin. Anal., **8** (2001), 27–36
- [2] Brézis, H., *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2010
- [3] Cabada, A., Precup, R., Saavedra, L. and Tersian, S., *Multiple positive solutions to a fourth order boundary value problem*, Electron. J. Differential Equations, 2016 (2016), No. 254, 1–18
- [4] Cabada, A. and Saavedra, L., *Existence of solutions for  $n^{\text{th}}$ -order nonlinear differential boundary value problem by means of new fixed points theorems*, Nonlinear Anal. Real World Appl., **42** (2018), 180–206
- [5] Chyan, C. J. and Henderson, J., *Positive solutions of  $2m$  th-order boundary value problems*, Appl. Math. Lett. **15** (2002), 767–774
- [6] Graef, J. R. and Henderson, J., *Double solutions of boundary value problems for  $2m$  th-order differential equations and difference equations*, Comput. Math. Appl., **45** (2003), 873–885
- [7] Graef, J. R., Qian, C. and Yang, B., *Multiple symmetric positive solutions of a class of boundary value problems for higher order ordinary differential equations*, Proc. Amer. Math. Soc., **131** (2003), 577–585
- [8] Liu, J., *Existence and uniqueness of positive solution for  $2m$  th-order nonlinear differential equation with boundary conditions*, J. Appl. Math. Phys., **3** (2015), 1178–1185
- [9] O'Regan, D. and Precup, R., *Theorems of Leray-Schauder Type and Applications*, Taylor and Francis, 2001
- [10] Precup, R., *A compression type mountain pass theorem in conical shells*, J. Math. Anal. Appl., **338** (2008), 1116–1130
- [11] Precup, R., *Critical point theorems in cones and multiple positive solutions of elliptic problems*, Nonlinear Anal., **75** (2012), 834–851
- [12] Shi, G. and Chen, S., *Positive solutions of even higher-order singular superlinear boundary value problems*, Comput. Math. Appl., **45** (2003), 593–603
- [13] Twaty, A. Al and Eloë, P. W., *Concavity of solutions of a  $2m$ -th order problem with symmetry*, Opuscula Math., **33** (2013), 603–613
- [14] Yang, X., *Green's function and positive solutions for higher-order ODE*, Appl. Math. Comput., **136** (2003), 379–393

FACULTY OF MATHEMATICS  
USTHB, PO. BOX 32, EL-ALIA BAB-EZZOUAR  
ALGIERS, ALGERIA  
E-mail address: habiba.boulaiki@yahoo.fr

Received: 04.03.2018; In revised form: 26.04.2018; Accepted: 01.05.2018

2010 Mathematics Subject Classification. 34B40, 47J30, 34B18.

Key words and phrases. Higher order boundary value problem, critical point, mountain pass geometry, compression-expansion, cone, positive solution, Harnack inequality.

Corresponding author: Radu Precup; r.precup@math.ubbcluj.ro

LABORATORY OF FIXED POINT THEORY AND APPLICATIONS  
ÉCOLE NORMALE SUPÉRIEURE  
KOUBA, ALGIERS, ALGERIA  
*E-mail address:* moussaoui@ens-kouba.dz

DEPARTMENT OF MATHEMATICS  
BABEȘ-BOLYAI UNIVERSITY  
CLUJ-NAPOCA, ROMANIA  
*E-mail address:* r.precup@math.ubbcluj.ro