

Approximation by generalized Stancu type integral operators involving Sheffer polynomials

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ABSTRACT.

In this article, we give a generalization of integral operators which involves Sheffer polynomials introduced by Sucu and Büyükyazıcı. We obtain approximation properties of our operators with the help of the universal Korovkin's theorem and study convergence properties by using modulus of continuity, the second order modulus of smoothness and Peetre's K -functional. We have also established Voronovskaja type asymptotic formula. Furthermore, we study the convergence of these operators in weighted spaces of functions on the positive semi-axis and estimate the approximation by using weighted modulus of continuity.

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