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Proximal point algorithms involving fixed point iteration for nonexpansive mappings in CAT(κ) spaces

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ABSTRACT.

In this paper, we propose a new modified proximal point algorithm involving fixed point iteration for nonexpansive mappings in CAT(1) spaces. Under some mild conditions, we prove that the sequence generated by our iterative algorithm Δ-converges to a common solution between certain convex optimization and fixed point problems.

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