

Existence of solutions of implicit integral equations via Z -contraction

PRADIP R. PATLE and DEEPESH KUMAR PATEL

ABSTRACT.

The main focus of this work is to assure that the sum of a compact operator with a Z -contraction admits a fixed point. The concept of condensing mapping (in the sense of Hausdorff non-compactness measure) is used to establish the concerned result which generalizes some of the existing state-of-art in the literature. Presented result is used to verify the actuality of solutions of implicit integral equations.

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Corresponding author: Deepesh Kumar Patel; deepesh456@gmail.com

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DEPARTMENT OF MATHEMATICS

VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY

NAGPUR, 440010 INDIA

E-mail address: pradip.patle12@gmail.com

E-mail address: deepesh456@gmail.com