

Solution of the singular Cauchy problem for a general inhomogeneous Euler–Poisson–Darboux equation

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ABSTRACT.

In this paper, we solve Cauchy problem for a general form of an inhomogeneous Euler–Poisson–Darboux equation, where Bessel operator acts instead of the each second derivative. In the classical formulation, the Cauchy problem for this equation is not correct. However, for a specially selected form of the initial conditions, the equation has a solution. The general form of the Euler–Poisson–Darboux equation with such conditions we will call the singular Cauchy problem.

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