

## Fixed point theorems for contractions in semicomplete semimetric spaces

TOMONARI SUZUKI

### ABSTRACT.

We introduce the concept of semicompleteness on semimetric space, which is weaker than completeness. We prove fixed point theorems for contractions in semicomplete semimetric spaces. Also, we generalize Jachymski-Matkowski-Świątkowski's fixed point theorem in semimetric spaces.

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DEPARTMENT OF BASIC SCIENCES  
KYUSHU INSTITUTE OF TECHNOLOGY  
FACULTY OF ENGINEERING,  
TOBATA, KITAKYUSHU 804-8550, JAPAN  
E-mail address: suzuki-t@mns.kyutech.ac.jp

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