

Dedicated to Professor Yeol Je Cho on the occasion of his retirement

Levitin-Polyak well-posedness for parametric quasivariational inclusion and disclusion problems

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ABSTRACT.

In this paper, we aim to suggest the new concept of Levitin-Polyak (for short, LP) well-posedness for the parametric quasivariational inclusion and disclusion problems (for short, (QVIP) (resp. (QVDP))). Necessary and sufficient conditions for LP well-posedness of these problems are proved. As applications, we obtained immediately some results of LP well-posedness for the quasiaequilibrium problems and for a scalar equilibrium problem.

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