

*Dedicated to Professor Yeol Je Cho on the occasion of his retirement*

## A new iterative method for the split feasibility problem

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### ABSTRACT.

The split feasibility problem (SFP) has many applications, which can be a model for many inverse problems where constraints are imposed on the solutions in the domain of a linear operator as well as in the operator's range. In this paper, we introduce a new projection method to solve the SFP and prove its convergence under standard assumptions. Our results improve previously known corresponding methods and results of this area. The preliminary numerical experiments illustrates the advantage of our proposed methods.

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