

Dedicated to Professor Yeol Je Cho on the occasion of his retirement

Some surjectivity results for operators of generalized monotone type via a topological degree

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ABSTRACT.

We introduce a topological degree for a class of operators of generalized monotone type in reflexive Banach spaces, based on the recent Berkovits degree. Using the degree theory, we give some surjectivity results for operators of generalized monotone type in reflexive Banach spaces. In the Hilbert space case, this reduces to the celebrated Browder-Minty theorem for monotone operators.

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