

Dedicated to Professor Yeol Je Cho on the occasion of his retirement

Existence and stability for a generalized differential mixed quasi-variational inequality

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ABSTRACT.

In the present paper, we investigate a generalized differential mixed quasi-variational inequality consisting of a system of an ordinary differential equation and a generalized mixed quasi-variational inequality. By using an important result concerning the measurable selection, we prove the existence of Carathéodory weak solution to the generalized differential mixed quasi-variational inequality. Then, with the existence result, we establish two stability results for the generalized differential mixed quasi-variational inequality under different conditions, i.e., upper semicontinuity and lower semicontinuity of the Carathéodory weak solution with respect to the parameter, which is a perturbation of some mappings in the generalized mixed quasi-variational inequality.

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