

Dedicated to Professor Yeol Je Cho on the occasion of his retirement

On existence of solution of a class of quadratic-integral equations using contraction defined by simulation functions and measure of noncompactness

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ABSTRACT.

In this paper we have introduced a new type of contraction condition using a class of simulation functions, in the sequel using the new contraction definition, involving measure of noncompactness; we establish few results on existence of fixed points of continuous functions defined on a subset of Banach space. This result also generalizes other related results obtained in Arab [Arab, R., *Some generalizations of Darbo fixed point theorem and its application*, Miskolc Math. Notes, **18** (2017), No. 2, 595–610], Banaś [Banaś, J. and Goebel, K., *Measures of Noncompactness in Banach Spaces*, Lecture Notes in Pure and Applied Mathematics, Dekker, New York, 60 (1980)]. The obtained results are used in establishing existence theorems for a class of nonlinear quadratic equation (which generalizes several types of fractional-quadratic integral equations such as Abel's integral equation) defined on a closed and bounded subset of \mathbb{R} . The existence of solution is established with the aid of a measure of noncompactness defined on function space $C(I)$ introduced in [Banaś, J. and Olszowy, L., *Measures of Noncompactness related to monotonicity*, Comment. Math., **41** (2001), 13–23].

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