

*Dedicated to Professor Yeol Je Cho on the occasion of his retirement*

## On the Krein-Milman theorem in $CAT(\kappa)$ spaces

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### ABSTRACT.

Let  $\kappa > 0$  and  $(X, \rho)$  be a complete  $CAT(\kappa)$  space whose diameter smaller than  $\frac{\pi}{2\sqrt{\kappa}}$ . It is shown that if  $K$  is a nonempty compact convex subset of  $X$ , then  $K$  is the closed convex hull of its set of extreme points. This is an extension of the Krein-Milman theorem to the general setting of  $CAT(\kappa)$  spaces.

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### REFERENCES

- [1] Bridson, M. and Haefliger, A., *Metric Spaces of Non-Positive Curvature*, Springer, Berlin, 1999
- [2] Brown, K. S., *Buildings*, Springer, New York, 1989
- [3] Cho, Y. J., Ciric, L. and Wang, S. H., *Convergence theorems for nonexpansive semigroups in  $CAT(0)$  spaces*, *Nonlinear Anal.*, **74** (2011), 6050–6059
- [4] Cholamjiak, P., Abdou, A. A. N. and Cho, Y. J., *Proximal point algorithms involving fixed points of nonexpansive mappings in  $CAT(0)$  spaces*, *Fixed Point Theory Appl.*, **2015**, 2015:227, 13 pp.
- [5] Krein, M. G. and Milman, D. P., *On extreme points of regular convex sets*, *Studia Math.*, **9** (1940), 133–137
- [6] Niculescu, C. P., *The Krein-Milman theorem in global NPC spaces*, *Bull. Math. Soc. Sci. Math. Roumanie.*, **50** (2007), 343–346
- [7] Ohta, S., *Convexities of metric spaces*, *Geom. Dedicata.*, **125** (2007), 225–250
- [8] Pakkaranang, N., Sa Ngiamsunthorn, P., Kumam, P. and Cho, Y. J., *Convergence theorems of the modified  $S$ -type iterative method for  $(a,)$ -generalized hybrid mappings in  $CAT(0)$  spaces*, *J. Math. Anal.*, **8** (2017), 103–112
- [9] Saipara, P., Chaipunya, P., Cho, Y. J. and Kumam, P., *On strong and  $\Delta$ -convergence of modified  $S$ -iteration for uniformly continuous total asymptotically nonexpansive mappings in  $CAT(\kappa)$  spaces*, *J. Nonlinear Sci. Appl.*, **8** (2015), 965–975

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