

Dedicated to Professor Yeol Je Cho on the occasion of his retirement

Hybrid Bregman projection methods for fixed point and equilibrium problems

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ABSTRACT.

The purpose of this article is to investigate a projection algorithm for solving a fixed point problem of a closed multi-valued Bregman quasi-strict pseudocontraction and an equilibrium problem of a bifunction. Strong convergence of the projection algorithm is obtained without any compact assumption in a reflexive Banach space. As applications, monotone variational inequality problems are considered. Finally, a numerical simulation example is presented for demonstrating the feasibility and convergence of the algorithm proposed in main result.

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