

*Dedicated to Professor Yeol Je Cho on the occasion of his retirement*

## A note on the accelerated proximal gradient method for nonconvex optimization

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### ABSTRACT.

We improve a recent accelerated proximal gradient (APG) method in [Li, Q., Zhou, Y., Liang, Y. and Varshney, P. K., *Convergence analysis of proximal gradient with momentum for nonconvex optimization*, in Proceedings of the 34th International Conference on Machine Learning, Sydney, Australia, PMLR 70, 2017] for nonconvex optimization by allowing variable stepsizes. We prove the convergence of the APG method for a composite nonconvex optimization problem under the assumption that the composite objective function satisfies the Kurdyka-Łojasiewicz property.

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